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Two Core Radii For Minimum Total Dispersion In Single-Mode Step-Index Optical Fibers

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***Abstract*—Starting from the operating wavelength and the chemical composition of the materials that integrate the core and cladding of an optical fiber, a method was developed for the calculation of the values of the core radii; it allows fiber operation in a monomode region with minimum total dispersion.**

The study is restricted to step-index fibers and the selected theoretical model is based on the weakly-guiding characteristic equation. From these considerations it is possible to obtain two different values of core radii for a given source operating wavelength.

The theory described allows the characterization of an optical fiber for use with a given light source and extends a previously described theory.

I. INTRODUCTION

The spread of light pulses transmitted through single-mode optical fibers is caused by two main factors, material dispersion and waveguide dispersion. The first factor results from the dependence of the refractive indexes of the materials used in the construction of the core and cladding of the optical fiber on the wavelength. The second factor takes into consideration the effect of the geometry of the guiding structure (the optical waveguide) on its fundamental mode. Both factors combine and the result is known as the total dispersion. It is worth mentioning that this combination does not result from the simple addition of the two factors mentioned above but rather it is much more complex than this [1], [2].

In order to reduce as much as possible the pulse spread and obtain as a consequence an increase in the operating passband available, optical fibers with minimum total dispersion at the source wavelength, $\lambda = \hat{\lambda}$, should be used. The value of $\hat{\lambda}$ is obtained through the solution of the total dispersion equation.

Various methods have been proposed for solving this problem. In the case of single-mode step-index optical fibers these methods are essentially based on three procedures: 1) the use of the weakly guiding characteristic equation [1], [4], [5]; 2) the use of asymptotic approximations for the eigenvalues of the weakly guiding characteristic equation [6], [8], [9]; and 3) the use of the exact characteristic equation [3].

Manuscript received May 30, 1985; revised December 2, 1985. This work was supported in part by CNPQ and Telebrás This work was performed at the State University of Campinas (UNICAMP), Campinas, SP, Brazil.

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For the calculation of the wavelength for minimum total dispersion $\hat{\lambda}$, using the exact characteristic equation, the complexity of the algorithms used and the large number of data to be manipulated [3] require computer systems of medium or large size. When the weakly guiding characteristic equation is used, the amount of data to be manipulated is reduced due to the relative simplicity of the equations and algorithms involved. When asymptotic approximations are used the computational procedures may be implemented on small programmable calculators. The use of any of these procedures will depend on the available computational system and the required precision of the results.

For the calculation of $\hat{\lambda}$ it is first necessary to have prior knowledge of the optical-fiber physical characteristics, such as the core radius and the chemical composition of the materials used for the construction of the fiber core and cladding. Once the value of $\hat{\lambda}$ is found, the most appropriate optical source to operate the fiber under minimum total dispersion may be chosen. This is an analysis procedure where, given an optical fiber, the optimum source to operate with the fiber can be found.

An opposite problem to the one just described, which consists of synthesizing an optical fiber for optimal operation with a given optical source, is usually of particular interest.

A method for accomplishing the synthesis of single-mode step-index optical fibers has been reported in a previous work [10]. Asymptotic approximations proposed by Miyagi and Nishida [8] were used in that work. The chemical composition of the core and cladding materials were assumed known and the available light source wavelength was chosen equal to $\hat{\lambda}$. In this case, the total dispersion equation is used for the calculation of the fiber core radius. From the characteristics of the method adopted, the calculated radius is the one that allows pulse transmission with minimum total dispersion when the fiber operates with the wavelength of the available source. Due to its simplicity, all the computational procedures were implemented on a small programmable calculator. However, due to the asymptotic approximation used, only one value of radius for minimum total dispersion was found. The existence of two core radii for the same value of $\hat{\lambda}$ was suggested in a previous work [3].

In the present work the weakly guiding characteristic equation was used as the theoretical basis for the synthesis of single-mode step-index optical fiber. The use of this equation allows for the calculation of the two-core radii that yield minimum total dispersion, as predicted in [3]. As expected, the use of the weakly guiding characteristic equation reduces the amount of data to be manipulated and the complexity of the algorithms to be adopted.

In the following sections, a description of the theory is given along with the results of a few cases.

II. BASIC EQUATIONS

The total dispersion equation which is the wavelength derivative of the transit time per unit fiber length, is given by [6]

$$D_T = (\lambda / c n_e)\left\{(1-b)\nu_2 + b\nu_1 + 2b'\phi + b''\theta/2 - (n_e)^{-2}\left[n_2 n_2' + b\phi + b'\theta/2\right]^2\right\} \quad (1)$$

where c and λ are the free-space phase velocity and wavelength of the light wave, respectively; b is the normalized propagation constant given by the relation

$$b = 1 - \frac{U^2}{V^2} = \frac{W^2}{V^2} \quad (2)$$

U and W are the eigenvalues obtained from the weakly guiding characteristic equation

$$\frac{UJ_1(U)}{J_0(U)} = \frac{WK_1(W)}{K_0(W)} \tag{3}$$

with

$$0 \leqq V = (U^2 + W^2)^{1/2} \leqq 2.40483 \tag{4}$$

and J and K represent the Bessel and modified Hankel functions, respectively. The parameter V is the normalized frequency given by

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} \tag{5}$$

where a is the core radius and n_1 and n_2 are the refractive indexes of the core and cladding materials, respectively. In addition

$$\nu_j = n_j n_j'' + (n_j')^2, \qquad j = 1,2 \tag{6a}$$

$$\phi = n_1 n_1' - n_2 n_2' \tag{6b}$$

$$\theta = n_1^2 - n_2^2 \tag{6c}$$

$$n_e^2 = n_2^2 + b\theta. \tag{6d}$$

The quantity n_e is the effective phase index. The prime (′) and double prime (″) are used to indicate the first and second differentiation with respect to the wavelength λ, respectively.

The nonlinear dependence of the refractive indexes on the wavelength is included by using the three-term Sellmeier equation

$$n_j^2 = 1 + \sum_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - l_i^2}, \qquad j = 1,2 \tag{7}$$

where A_i are constants related to the number of particles in the material that can oscillate at wavelengths l_i. The derivatives of the refractive indexes for use in (6) are obtained from (7) and given by

$$n_j' = \frac{1}{n_j} \sum_{i=1}^{3} \frac{A_i l_i^2 \lambda}{(\lambda^2 - l_i^2)^2}, \qquad j = 1,2 \tag{8}$$

$$n_j'' = \frac{1}{n_j}\left[-(n_j')^2 + \sum_{i=1}^{3} \frac{A_i l_i^3 (3\lambda^2 + l_i^2)}{(\lambda^2 - l_i^2)^3} \right], \qquad j = 1,2. \tag{9}$$

The first and second derivatives of the normalized propagation constant b, may be obtained from (2) and (3) with the result [4]

$$b' = \frac{2U^2}{V^2} \cdot \frac{K_0^2(W)}{K_1^2(W)} A \tag{10}$$

$$b'' = \frac{2U^2}{V^2} \cdot \frac{K_0^2(W)}{K_1^2(W)} B + (VA)^2 \left[\frac{4U^2}{V^3} \left(-\frac{K_0(W)}{K_1(W)} + \frac{K_0^2(W)}{WK_1^2(W)} + \frac{K_0^3(W)}{K_1^3(W)} \right) \left(\frac{W}{V} + \frac{U^2}{WV} \frac{K_0^2(W)}{K_1^2(W)} \right) - \frac{2U^2}{V^4} \left(1 + \frac{2K_0^2(W)}{K_1^2(W)} \right) \frac{K_0^2(W)}{K_1^2(W)} \right] \tag{11}$$

TABLE I
VALUES OF RADII OF MONOMODE OPTICAL FIBERS WITH A STEP-INDEX PROFILE FOR PULSE TRANSMISSION WITH MINIMUM TOTAL DISPERSION

Fiber	Operating Wavelength (μm)	Core radii and V-value for minimum total dispersion (μm)	
A	λ = 1.7542	$\hat{a}$ = 0.9622	$\hat{a}$ = 1.7500
		V = 0.8807	V = 1.6018
B	λ = 1.55	$\hat{a}$ = 1.2696	$\hat{a}$ = 2.0131
		V = 0.9345	V = 1.4818

Note: Core material of fiber A: 13.5-percent GeO_2–86.5-percent SiO_2; core material of fiber B: 7.0-percent GeO_2–93.0-percent SiO_2

where

$$A = (1/\lambda) - (\phi/\theta) \tag{12}$$

$$B = (\nu_1 - \nu_2)/\theta - (\phi/\theta)^2 + (2A/\lambda). \tag{13}$$

The two possible values of the core radius for minimum total dispersion are obtained from (1) for a given value of $\hat{\lambda}$. Thus the problem is reduced to the solution of the equation

$$D_T(\hat{\lambda} = \lambda_{\mathrm{oper}})\big|_{a=\hat{a}} = 0 \tag{14}$$

where the symbol (ˆ) is used to indicate the value for minimum total dispersion. The two possible solutions for $\hat{a}$, a_1, and a_2, are called the core radii for minimum total dispersion.

III. Numerical Results

In Table I, a few pairs of core radii of optical fibers for pulse transmission with minimum total dispersion at a given wavelength are shown. The results shown are for two selected fibers. These values were obtained by means of a computer program, with a precision of up to 10^{-6}. For both fibers considered, the cladding material is fused SiO_2.

The behavior of the total dispersion as a function of the core radius of the fiber at a given operating wavelength is shown in Figs. 1 and 2. Note in these figures the pronounced dispersion when only the core material is changed. This shows the influence that the waveguide dispersion has on the choice of the core radius for transmission with minimum total dispersion. This influence has already been investigated in the case of using an asymptotic approximation for the weakly guiding characteristic equation [10].

IV. Conclusion

A method based on the weakly guiding approximation useful for the synthesis of single-mode step-index optical fibers for operation with minimum total dispersion was presented. This method is an extension of a previously presented one [10] and allows the calculation of the second value of the fiber core radius, not possible before. This method also served to verify that the wave guide dispersion has a large influence on the calculated values of core radii as shown here. However, it is important to note that the smaller optimum core-diameter values might be impractical because in such cases the field will spread to the cladding region and bending-induced radiation loss will increase.

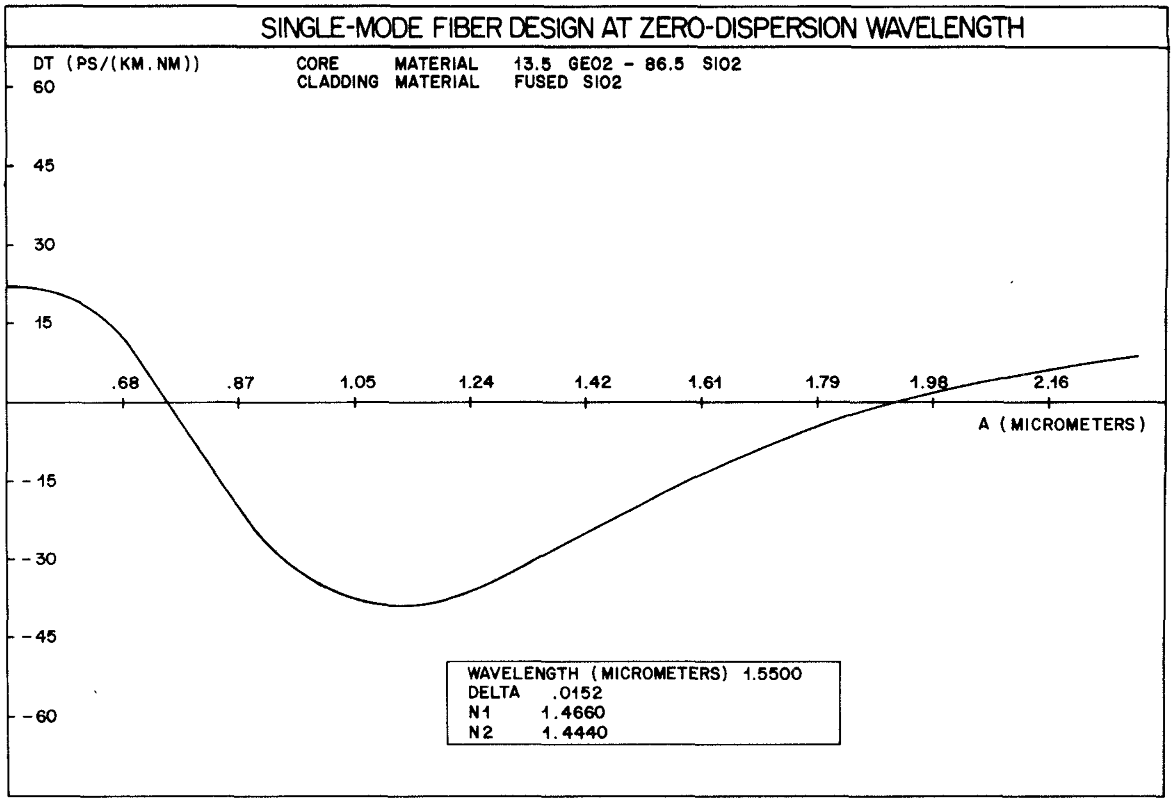

Fig 1 Synthesis of monomode fibers with step-index profile, implemented in a desk calculator $\lambda = 1.55\ \mu m$. Core material: 13.5-percent GeO_2–86 5-percent SiO_2. Cladding material: fused SiO_2.

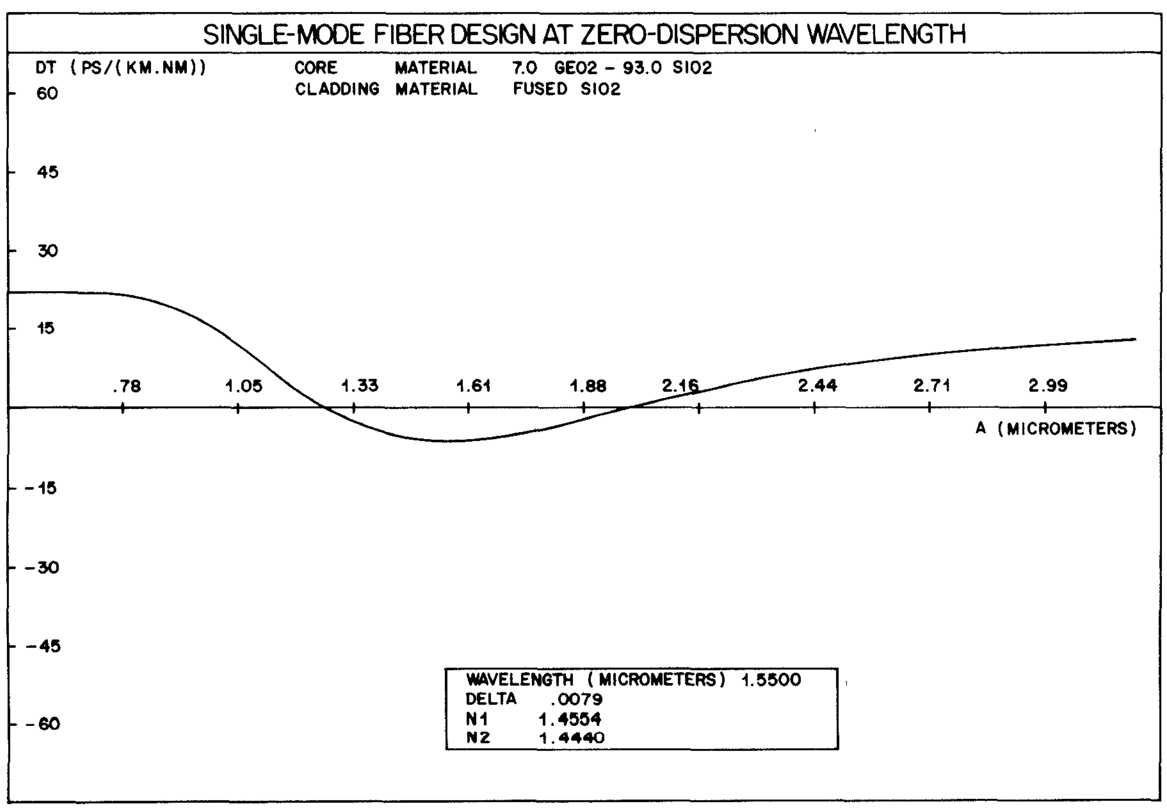

Fig. 2. Same as in Fig. 1, with a core material: 7.0-percent GeO_2–93.0-percent SiO_2.

Acknowledgment

The authors are thankful for the valuable discussions held with Prof. Dr. David A. Rogers from North Dakota State University, Fargo, ND.

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On the Measurement of Noise Parameters of Microwave Two-Ports

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***Abstract*—A novel procedure for determining the noise parameters of microwave two-ports is introduced. In this procedure, the computations necessary to find the noise parameters from the set of measurements of noise temperature (noise figure) are greatly simplified. The assessment of accuracy with which the noise parameters can be determined from a given set of measurement data is straightforward.**

I. Introduction

A typical noise parameter measurement setup is schematically shown in Fig. 1. The noise parameters of a device under test (DUT) and those of a receiver are represented by pairs of noise sources having correlation matrices [1]–[5]

$$[C_D] = \begin{bmatrix} \overline{e_{nD}^2} & \overline{e_{nD} i_{nD}^*} \\ \overline{e_{nD}^* i_{nD}} & \overline{i_{nD}^2} \end{bmatrix} \tag{1}$$

$$[C_R] = \begin{bmatrix} \overline{e_{nR}^2} & \overline{e_{nR} i_{nR}^*} \\ \overline{e_{nR}^* i_{nR}} & \overline{i_{nR}^2} \end{bmatrix} \tag{2}$$

Manuscript received June 5, 1985; revised November 14, 1985.

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IEEE Log Number 8407189

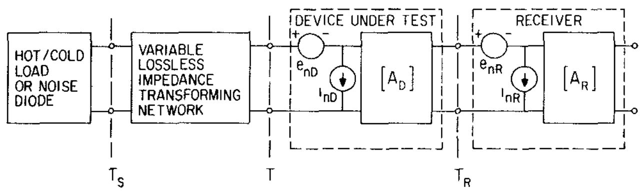

Fig. 1. A typical noise parameter measurement setup.

respectively. These matrices have to be Hermitian and nonnegative definite [3]–[5]. If the signal parameters of a DUT are given by chain matrix $[A_D]$, then the noise parameters of the cascade connection of the DUT and receiver given in a form of correlation matrix $[C]$ are [5]

$$[C] = [C_D] + [A_D][C_R][A_D]^{\dagger} \tag{3}$$

where the "dagger" designates the complex conjugate of the transpose of $[A_D]$ matrix. Matrix $[C]$ represents the noise parameters that can be determined at plane T (Fig. 1) by at least four noise temperature (noise figure) measurements for different values of source impedance as provided by the impedance transforming network. It is clear that if the noise parameters of a DUT are desired, the receiver contribution can be removed using (3), provided receiver noise parameters and device signal parameters are known.

The noise temperature T_n of any linear two-port is most commonly written in the following form [1], [2]:

$$T_n = T_{\min} + NT_0 \frac{|Z_s - Z_{\mathrm{opt}}|^2}{R_s R_{\mathrm{opt}}}$$

$$= T_{\min} + NT_0 \frac{|Y_s - Y_{\mathrm{opt}}|^2}{G_s G_{\mathrm{opt}}} \tag{4}$$

where

$$N = G_n R_{\mathrm{opt}} = R_n G_{\mathrm{opt}} \tag{5}$$

and

$T_{\min}$	minimum noise temperature,
$T_0 = 290$ K	standard temperature,
$Z_s = R_s + jX_s$	source impedance,
$Y_s = G_s + jB_s$	source admittance,
$Z_{\mathrm{opt}} = R_{\mathrm{opt}} + jX_{\mathrm{opt}}$	optimum source impedance,
$Y_{\mathrm{opt}} = G_{\mathrm{opt}} + jB_{\mathrm{opt}}$	optimum source admittance,
R_n	noise resistance,
G_n	noise conductance.

$T_{\min}$, R_{opt}, X_{opt}, G_n and $T_{\min}$, G_{opt}, B_{opt}, R_n are the sets of noise parameters equivalent to the correlation matrix $[C]$ (appropriate relations are given, for instance, in [2]). It has been shown that both $T_{\min}$ and the parameter N are invariant under transformation through lossless reciprocal two-ports connected to the input of a noisy two-port [2]. It also has been observed that for $T_{\min}$ and N to represent a physical two-port, the following inequality has to be satisfied [6]:

$$T_{\min} \leqslant 4NT_0. \tag{6}$$

This inequality (together with rather obvious conditions: $T_{\min} \geqslant 0, G_n \geqslant 0$ (or $R_n \geqslant 0$)) follows directly from the property that the correlation matrices have to be Hermitian and nonnegative definite. A simple physical interpretation of this inequality is